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# Loop corrections to sub-leading heavy quark currents in SCET

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## Abstract

We compute the one-loop (hard) matching correction to heavy-to-light transition currents in soft-collinear effective theory (SCET) to sub-leading power in the SCET expansion parameter for an arbitrary Dirac structure of the QCD weak current.

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# 1 Introduction

The theoretical description of exclusive or semi-inclusive  $B$  meson decays into final states consisting of light particles is currently a topic of high interest. What distinguishes these decays from inclusive decays or decays into heavy (charmed) mesons, where the heavy quark expansion and heavy quark effective theory are useful, is the detection of particles or jets with small invariant mass compared to their large energy of order of  $m_b$ , the  $b$  quark mass. The appropriate theoretical framework [1] now involves factorization formulae similar to those justifying the use of perturbative QCD in high-energy collisions [2]. The corresponding equations can be derived in a transparent way with soft-collinear effective theory (SCET) [3, 4, 5].

The weak currents  $\bar{\psi}\Gamma Q$  constitute a primary source of flavour-changing transitions in semi-leptonic  $B$  decays, and less directly also in radiative and non-leptonic decays. ( $Q$  denotes a heavy quark field,  $\psi$  a light quark field, and  $\Gamma$  a Dirac structure.) The accurate representation of these currents in the effective theory is therefore an important problem. In this paper we are concerned with the “large-recoil” region, where large momentum is transferred to the final state. After integrating out the short-distance modes of the strong interaction with virtuality  $m_b^2$ , the currents are represented in SCET as

$$\bar{\psi}\Gamma Q = \sum_i \tilde{C}_i^{(0)} \star J_i^{(0)} + \sum_k \tilde{C}_k^{(1)} \star J_k^{(1)} + \dots, \quad (1)$$

which realizes an expansion in the strong coupling  $\alpha_s(m_b)$  and the SCET expansion parameter  $\lambda$  of order  $(\Lambda_{\text{QCD}}/m_b)^{1/2}$ . In this equation the  $C$ ’s denote the short-distance coefficients of the leading and sub-leading currents in powers of  $\lambda$ ,  $J_i^{(0)}$  and  $J_k^{(1)}$ , respectively. (The notation will be made more precise below; the asterisk indicates that the product involves convolutions, which is typical of the non-local nature of SCET that contains dynamical collinear modes with some momentum components of order  $m_b$ .)

The coefficient functions of the leading-power currents have been computed including one-loop corrections a few years ago in two different factorization schemes [3, 6]. In this paper we calculate the coefficients of all  $\lambda$ -suppressed currents to one-loop accuracy. The motivation for this is that SCET allows us to formulate factorization formulae at the level of power-suppressed effects, where these coefficients are needed. Furthermore and more important, it has recently become clear [7, 8] that the sub-leading currents we consider here contribute leading effects to semi-leptonic decays due to a suppression of the matrix elements of leading-power currents. The one-loop corrections to the  $J_k^{(1)}$  are one of two effects that need to be computed to evaluate the symmetry relations among heavy-to-light form factors to higher accuracy than currently available [6].

The outline of the paper is as follows: In Section 2 we introduce notation and the operator basis for the leading and sub-leading SCET current operators. In Section 3 we present some details of the matching calculation by example of the vector current. The results for the coefficient functions for an arbitrary Dirac structure of the original weak current are summarized in Section 4. We conclude in Section 5.

## 2 Notation and operator basis

SCET contains fields for soft fluctuations with momentum  $k \sim m_b \lambda^2 \sim \Lambda_{\text{QCD}}$  and hard-collinear fields for modes with large momentum in the direction of the light-like vector  $n_-$ . A hard-collinear momentum is decomposed as  $p = n_+ p n_- / 2 + p_\perp + n_- p n_+ / 2$  with component scaling

$$n_+ p \sim m_b, \quad p_\perp \sim m_b \lambda, \quad n_- p \sim m_b \lambda^2, \quad (2)$$

and  $n_\pm^2 = 0$ ,  $n_- n_+ = 2$ . We refer to [5] for further notation and conventions, but note the following change of terminology: we now call soft (hard-collinear) what has been called ultrasoft (collinear) in [5].

The part of the SCET Lagrangian involving quark fields at leading order in the expansion in  $\lambda$  reads [3]

$$\mathcal{L}^{(0)} = \bar{\xi} \left( i n_- D + i \not{D}_\perp c \frac{1}{i n_+ D_c} i \not{D}_\perp c \right) \frac{\not{n}_+}{2} \xi + \bar{q} i \not{D}_s q + \bar{h}_v i v D_s h_v. \quad (3)$$

The hard-collinear quark field  $\xi$  satisfies  $\not{n}_- \xi = 0$ ,  $q$  denotes the soft light quark. The soft heavy quark field describing fluctuations around the heavy quark momentum  $m_b v$  is  $h_v$  with  $\not{v} h_v = h_v$ . The index on the covariant derivative means that only a soft or hard-collinear field is included. Since in this paper we consider power corrections of order  $\lambda$ , we mention that the order  $\lambda$  corrections to the Lagrangian, not written in (3), all involve a single transverse hard-collinear vector. We also recall that soft fields are multipole-expanded in products with hard-collinear fields (see [5] for the corresponding details).

Without loss of generality we assume a reference frame with  $v_\perp = 0$ , implying  $n_+ v = 1/n_- v$ . The form of the SCET current operators is then restricted by hard-collinear and soft gauge invariance and invariance under the boost transformation  $n_- \rightarrow \alpha n_-$ ,  $n_+ \rightarrow n_+/\alpha$ . We consider transitions to energetic final states, which requires quark bilinears  $\bar{\xi}[\dots]h_v$  for the leading and sub-leading currents. Other possible field combinations are further suppressed by powers of  $\lambda$ . The projection properties of the quark fields then imply that there are only three independent Dirac structures, which we choose as  $\Gamma'_j = \{1, \gamma_5, \gamma'_\perp\}$ . Gauge-invariant currents are constructed from building blocks invariant under collinear gauge transformations. Up to order  $\lambda$  we have

$$\bar{\xi} W_c, \quad h_v, \quad [W_c^\dagger i D_\perp^\mu W_c] \quad (4)$$

at our disposal. ( $W_c$  is a hard-collinear Wilson line involving only the  $n_+ A_c$  component of the gluon field [5].) The leading-power currents are constructed from the first two invariants. The third is the only possible new structure at order  $\lambda$ .

Altogether this allows the following three sets of operators:

$$\begin{aligned} O_j^{(A0)}(s; x) &\equiv (\bar{\xi} W_c)(x + s n_+) \Gamma'_j h_v(x_-) \equiv (\bar{\xi} W_c)_s \Gamma'_j h_v, \\ O_{j\mu}^{(A1)}(s; x) &\equiv (\bar{\xi} i \overleftarrow{D}_{\perp c\mu} (i n_- v n_+ \overleftarrow{D}_c)^{-1} W_c)_s \Gamma'_j h_v, \\ O_{j\mu}^{(B1)}(s_1, s_2; x) &\equiv \frac{1}{m_b} (\bar{\xi} W_c)_{s_1} (W_c^\dagger i D_{\perp c\mu} W_c)_{s_2} \Gamma'_j h_v. \end{aligned} \quad (5)$$

Here  $O_j^{(A0)}$  are leading-power currents, and  $O_{j\mu}^{(A1)}$  and  $O_{j\mu}^{(B1)}$  are order  $\lambda$ . The operators are non-local in the  $n_+$  direction, because the  $n_+p$  component of a hard-collinear momentum  $p$  is of the same order as the hard fluctuations integrated out in matching SCET to QCD. In (5) we used a short-hand notation to denote the position argument of blocks of fields, and  $x_- = (n_+x)n_-/2$  is the position of the multipole-expanded heavy quark field. The use of  $(in_-vn_+\overleftarrow{D}_c)^{-1}$  rather than  $1/m_b$  in  $O_{j\mu}^{(A1)}$  to restore mass dimension three to the operator is a matter of convenience, since it makes the tree-level coefficient functions simple. Another reasonable choice would be  $\not{n}_+/2(in_+\overleftarrow{D}_c)^{-1}$  instead of  $(in_-vn_+\overleftarrow{D}_c)^{-1}$ , but we choose the latter because of its simpler Dirac structure. According to the number of independent position arguments, we also refer to  $O_j^{(A0)}$  and  $O_{j\mu}^{(A1)}$  as two-body currents, and to  $O_{j\mu}^{(B1)}$  as three-body currents [9]. Not listed in (5) are the order  $\lambda$  operators  $(\bar{\xi}W_c)(x+sn_+)\Gamma'_j x_{\perp\mu} D_s^\mu h_v(x_-)$ , which arise from the multipole expansion of the heavy quark field. The coefficient functions of these operators equal those of the corresponding leading-power operators  $O_j^{(A0)}$ . We therefore do not consider them further.

Including the dimensionless short-distance coefficients, the QCD weak currents  $\bar{\psi}\Gamma_i Q$  are represented in SCET as

$$\begin{aligned} (\bar{\psi}\Gamma_i Q)(0) = & \int d\hat{s} \sum_j \tilde{C}_{ij}^{(A0)}(\hat{s}) O_j^{(A0)}(s; 0) \\ & + \int d\hat{s} \sum_j \tilde{C}_{ij\mu}^{(A1)}(\hat{s}) O_j^{(A1)\mu}(s; 0) \\ & + \int d\hat{s}_1 d\hat{s}_2 \sum_j \tilde{C}_{ij\mu}^{(B1)}(\hat{s}_1, \hat{s}_2) O_j^{(B1)\mu}(s_1, s_2; 0) + \cdots, \end{aligned} \quad (6)$$

where the ellipses stand for  $\lambda^2$ -suppressed terms (not considered in this paper), and we defined the boost-invariant and dimensionless convolution variables  $\hat{s}_i \equiv sm/n_-v$ . The factorization formulae, where these coefficient functions are needed, are usually formulated in terms of convolutions in momentum fraction rather than position space convolutions. The momentum space coefficient functions are related to those defined above by

$$C(n_-vn_+p_i/m_b) = \int \prod_i d\hat{s}_i \tilde{C}(\hat{s}_i) e^{in_-v \sum_i \hat{s}_i n_+p_i/m_b}. \quad (7)$$

The actual matching calculation is also done in momentum space and yields the momentum space coefficient functions directly. At order  $\lambda$  there is also a time-ordered product term

$$i \int d^4y \int d\hat{s} \sum_j \tilde{C}_{ij}^{(A0)}(\hat{s}) T \left( O_j^{(A0)}(s; 0), \mathcal{L}_{\text{SCET}}^{(1)}(y) \right) \quad (8)$$

of the leading currents with the sub-leading terms of the SCET Lagrangian.

For any given Dirac structure  $\Gamma_i$  of the QCD weak current the Lorentz tensor coefficient functions in (6) are decomposed into scalar functions using the boost-invariant objects  $n_{-\mu}/n_-v$ ,  $v_\mu$ ,  $g_{\mu\nu}$  and  $\epsilon_{\mu\nu\rho\sigma}$ . The tensor structures are then multiplied with the operator, which for each  $\Gamma_i$  results in a basis of operators with scalar coefficient functions. The bases are listed below. Here we drop the position indices  $s_{1,2}$ , which should be clear from (5).

- Scalar current  $J = \bar{\psi}Q$ :

$$\begin{aligned}
J^{(A0)} &= (\bar{\xi}W_c)h_v \\
J^{(A1)} &= -(\bar{\xi}i\overleftarrow{\mathcal{D}}_{\perp c}(in_-vn_+\overleftarrow{D}_c)^{-1}W_c)h_v, \\
J^{(B1)} &= \frac{1}{m_b}(\bar{\xi}W_c)(W_c^\dagger i \not{D}_{\perp c}W_c)h_v
\end{aligned} \tag{9}$$

- Pseudo-scalar current  $J_5 = \bar{\psi}\gamma_5 Q$ :

$$\begin{aligned}
J_5^{(A0)} &= (\bar{\xi}W_c)\gamma_5 h_v \\
J_5^{(A1)} &= (\bar{\xi}i\overleftarrow{\mathcal{D}}_{\perp c}(in_-vn_+\overleftarrow{D}_c)^{-1}W_c)\gamma_5 h_v, \\
J_5^{(B1)} &= \frac{1}{m_b}(\bar{\xi}W_c)(W_c^\dagger i \not{D}_{\perp c}W_c)\gamma_5 h_v
\end{aligned} \tag{10}$$

- Vector current  $J_\mu = \bar{\psi}\gamma_\mu Q$ :

$$\begin{aligned}
J_\mu^{(A0)1-3} &= (\bar{\xi}W_c)\{\gamma_{\perp\mu}, \frac{n_{-\mu}}{n_-v}, v_\mu\}h_v \\
J_\mu^{(A1)1-3} &= (\bar{\xi}i\overleftarrow{\mathcal{D}}_{\perp c}(in_-vn_+\overleftarrow{D}_c)^{-1}W_c)\{\gamma_{\perp\mu}, \frac{n_{-\mu}}{n_-v}, -2v_\mu\}h_v, \\
J_\mu^{(A1)4} &= (\bar{\xi}i\overleftarrow{D}_{\perp c\mu}(in_-vn_+\overleftarrow{D}_c)^{-1}W_c)h_v, \\
J_\mu^{(B1)1-3} &= \frac{1}{m_b}(\bar{\xi}W_c)(W_c^\dagger i \not{D}_{\perp c}W_c)\{\gamma_{\perp\mu}, -\frac{n_{-\mu}}{n_-v}, v_\mu\}h_v \\
J_\mu^{(B1)4} &= \frac{1}{m_b}(\bar{\xi}W_c)(W_c^\dagger i D_{\perp c\mu}W_c)h_v
\end{aligned} \tag{11}$$

- Axial current  $J_{\mu 5} = \bar{\psi}\gamma_\mu\gamma_5 Q$ :

$$\begin{aligned}
J_{\mu 5}^{(A0)1-3} &= (\bar{\xi}W_c)\{\gamma_{\perp\mu}, -\frac{n_{-\mu}}{n_-v}, v_\mu\}\gamma_5 h_v \\
J_{\mu 5}^{(A1)1-3} &= (\bar{\xi}i\overleftarrow{\mathcal{D}}_{\perp c}(in_-vn_+\overleftarrow{D}_c)^{-1}W_c)\{-\gamma_{\perp\mu}, \frac{n_{-\mu}}{n_-v}, -2v_\mu\}\gamma_5 h_v, \\
J_{\mu 5}^{(A1)4} &= (\bar{\xi}i\overleftarrow{D}_{\perp c\mu}(in_-vn_+\overleftarrow{D}_c)^{-1}W_c)\gamma_5 h_v, \\
J_{\mu 5}^{(B1)1-3} &= \frac{1}{m_b}(\bar{\xi}W_c)(W_c^\dagger i \not{D}_{\perp c}W_c)\{\gamma_{\perp\mu}, -\frac{n_{-\mu}}{n_-v}, v_\mu\}\gamma_5 h_v \\
J_{\mu 5}^{(B1)4} &= \frac{1}{m_b}(\bar{\xi}W_c)(W_c^\dagger i D_{\perp c\mu}W_c)\gamma_5 h_v
\end{aligned} \tag{12}$$

- Tensor current  $J_{\mu\nu} = \bar{\psi}i\sigma_{\mu\nu}Q$ :

$$J_{\mu\nu}^{(A0)1-4} = (\bar{\xi}W_c)\{i\sigma_{\mu\perp\nu\perp}, \frac{n_{-[\mu}\gamma_{\perp\nu]}}{n_-v}, v_{[\mu}\gamma_{\perp\nu]}, -\frac{n_{-[\mu}v_{\nu]}}{n_-v}\}h_v$$

$$\begin{aligned}
J_{\mu\nu}^{(A1)1-3} &= (\bar{\xi} i \overleftarrow{\mathcal{D}}_{\perp c} (in_- v n_+ \overleftarrow{D}_c)^{-1} W_c) \left\{ -\frac{n_{-[\mu} \gamma_{\perp\nu]}}{n_- v}, 2v_{[\mu} \gamma_{\perp\nu]}, -\frac{n_{-[\mu} v_{\nu]}}{n_- v} \right\} h_v \\
J_{\mu\nu}^{(A1)4-6} &= (\bar{\xi} i \overleftarrow{D}_{\perp c} [\mu \gamma_{\perp\nu}], \frac{n_{-\nu]}{n_- v}, v_{\nu}] (in_- v n_+ \overleftarrow{D}_c)^{-1} W_c) h_v, \\
J_{\mu\nu}^{(A1)7} &= \frac{1}{2} (\bar{\xi} i \overleftarrow{\mathcal{D}}_{\perp c} (in_- v n_+ \overleftarrow{D}_c)^{-1} W_c) \gamma_{\perp[\mu} \gamma_{\perp\nu]} h_v - J_{\mu\nu}^{(A1)4}, \\
J_{\mu\nu}^{(B1)1-3} &= \frac{1}{m_b} (\bar{\xi} W_c) (W_c^\dagger i \not{D}_{\perp c} W_c) \left\{ \frac{n_{-[\mu} \gamma_{\perp\nu]}}{n_- v}, v_{[\mu} \gamma_{\perp\nu]}, \frac{n_{-[\mu} v_{\nu]}}{n_- v} \right\} h_v \\
J_{\mu\nu}^{(B1)4-6} &= \frac{1}{m_b} (\bar{\xi} W_c) (W_c^\dagger i D_{\perp c} [\mu \gamma_{\perp\nu}], \frac{2n_{-\nu]}{n_- v}, v_{\nu}] W_c) h_v \\
J_{\mu\nu}^{(B1)7} &= \frac{1}{2m_b} (\bar{\xi} W_c) (W_c^\dagger i \not{D}_{\perp c} W_c) \gamma_{\perp[\mu} \gamma_{\perp\nu]} h_v - J_{\mu\nu}^{(B1)4}
\end{aligned} \tag{13}$$

Here  $a_{[\mu} b_{\nu]} = a_\mu b_\nu - a_\nu b_\mu$ . The operators  $J_{\mu\nu}^{(A1)7}$  and  $J_{\mu\nu}^{(B1)7}$  vanish in four dimensions, but must be kept since we regularize dimensionally.

We introduced signs and factors of 2 in the definition of the operators such that the momentum space coefficient functions at tree level are either 1 or 0. The full expression for the SCET current is

$$J_X = \sum_i \tilde{C}_X^{(A0)i} \star J_X^{(A0)i} + \sum_k \left\{ \tilde{C}_X^{(A1)k} \star J_X^{(A1)k} + \tilde{C}_X^{(B1)k} \star J_X^{(B1)k} \right\} + \dots, \tag{14}$$

which defines the coefficient functions for the scalar ( $X = S$ ), pseudo-scalar ( $P$ ), vector ( $V$ ), axial ( $A$ ) and tensor ( $T$ ) currents. Here the product of coefficient function and operator in coordinate space means a convolution over the arguments  $\hat{s}_i$  as in (6).

The coefficients of the sub-leading currents have been determined at tree-level in [5]. (See [10] for an earlier but incomplete discussion.) The operator basis has been constructed for the general case of  $v_\perp \neq 0$  in [9], which allows many more operators, whose coefficients are, however, not independent. After imposing  $v_\perp = 0$  the operator basis in [9] is consistent with the one above though the choice of basis operators is different. In [9] it was also shown that the coefficients of the two-body “A1” currents are expressed through those of the leading-power currents by reparameterization invariance, a result that we reproduce by different means below. (The basis of the tensor two-body operators in [9] has only six elements instead of the seven “A1” currents above, because the coefficient of the seventh operator vanishes identically in the basis chosen in [9] due to reparameterization invariance. In fact, exploiting these relations the basis could be reduced to only four “A1”-type operators.) The main result below is therefore the one-loop computation of the coefficients of the three-body “B1” currents.

### 3 Method of calculation

We now explain some technical aspects of the coefficient function calculation taking the vector current  $J_\mu = \bar{\psi} \gamma_\mu Q$  for illustration.

The coefficients of the two-body currents follow from the computation of the matrix element  $\langle q(p') | \bar{\psi} \gamma_\mu Q | b(p) \rangle$  of the renormalized vector current. (The vector current is conserved, but in general we assume that the currents are renormalized in the modified minimal subtraction ( $\overline{\text{MS}}$ ) scheme. The subtraction scale of the QCD weak current is denoted by  $\nu$  to distinguish it from the QCD/SCET factorization scale  $\mu$ .) The matrix element is decomposed into invariant form factors,

$$\langle q(p') | \bar{\psi} \gamma_\mu Q | b(p) \rangle = F_1 \bar{u}(p') \gamma_\mu u(p) + F_2 \bar{u}(p') \frac{p_\mu}{m_b} u(p) + F_3 \bar{u}(p') \frac{m_b p'_\mu}{p \cdot p'} u(p). \quad (15)$$

With  $p^2 = m_b^2$  and  $p'^2 = 0$  the form factors can only depend on the dimensionless ratio

$$\frac{2p \cdot p'}{m_b^2} = \frac{n_- v n_+ p'}{m_b} + \mathcal{O}(\lambda^2) \quad (16)$$

and logarithms of  $\mu/m_b$  or  $\nu/m_b$ . It is immediately clear from this that the coefficient functions of two-body currents at any order in  $\lambda$  are related to the form factors  $F_i$  and their derivatives. Only  $F_1$  is non-zero at tree-level,  $F_1 = 1 + \mathcal{O}(\alpha_s)$ ,  $F_{2,3} = \mathcal{O}(\alpha_s)$ .

To order  $\lambda$  we can replace  $2p \cdot p'/m_b^2$  by  $x \equiv n_- v n_+ p'/m_b$  in the argument of the form factors. The full light and heavy quark spinors have the decomposition

$$\bar{u}(p') = \bar{u}_c(p') \left( 1 - \frac{\not{p}'_\perp \not{n}_+}{n_+ p'} \right), \quad (17)$$

$$u(p) = \left( 1 + \frac{\not{k}}{2m_b} + \dots \right) u_v = u_v + \mathcal{O}(\lambda^2 u_v), \quad (18)$$

where the collinear and heavy quark spinors satisfy  $\not{n}_- u_c(p') = 0$  and  $\not{p} u_v = u_v$ , respectively, and  $p = m_b v + k$ . The first equation is exact and the second shows that we can replace  $u(p)$  by  $u_v$  to order  $\lambda$ . Inserting this into (15) and performing some Dirac algebra to reduce the result to structures matching the definition of the basis operators (11), we obtain

$$\begin{aligned} \langle q(p') | \bar{\psi} \gamma_\mu Q | b(p) \rangle &= F_1 \bar{u}_c(p') \gamma_{\perp\mu} u_v + (F_1 + F_3) \bar{u}_c(p') \frac{n_{-\mu}}{n_- v} u_v + F_2 \bar{u}_c(p') v_\mu u_v \\ &+ F_1 \bar{u}_c(p') \frac{\not{p}'_\perp}{n_- v n_+ p'} \gamma_{\perp\mu} u_v + (F_1 - F_3) \bar{u}_c(p') \frac{\not{p}'_\perp}{n_- v n_+ p'} \frac{n_{-\mu}}{n_- v} u_v \\ &+ \left( F_1 + \frac{F_2}{2} \right) \bar{u}_c(p') \frac{\not{p}'_\perp}{n_- v n_+ p'} (-2v_\mu) u_v + 2F_3 \bar{u}_c(p') \frac{p'_{\perp\mu}}{n_- v n_+ p'} u_v. \end{aligned} \quad (19)$$

The form factors have infrared divergences which we regulate dimensionally in  $d = 4 - 2\epsilon$  space-time dimensions. With this regulator all SCET loop diagrams vanish, since there are no small invariants the loop diagrams could depend on, and scaleless integrals are zero in dimensional regularization. Hence the  $b \rightarrow q$  matrix elements of the SCET currents take their tree-level values multiplied by a operator renormalization constant matrix. The

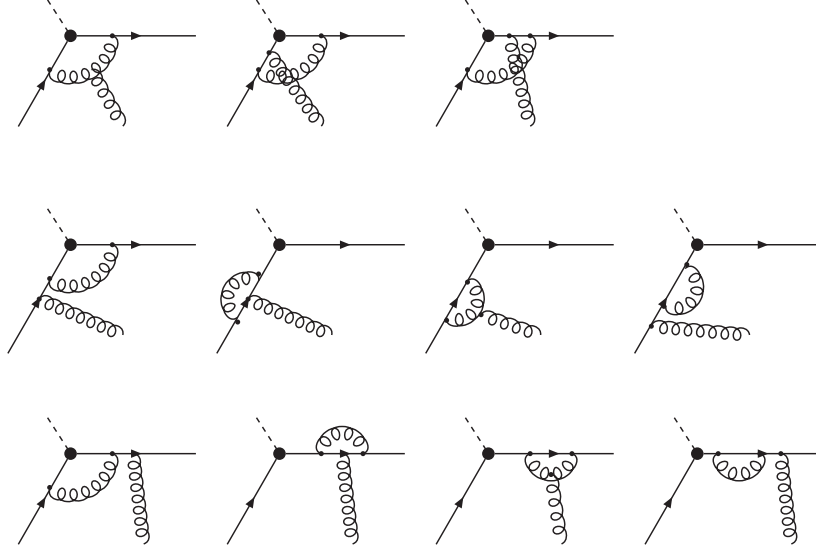


Figure 1: One-loop contributions to  $\langle q(p'_1)g(p'_2)|J_X|b(p)\rangle$ . Counterterm diagrams are not shown.

unrenormalized coefficients  $C_V^{(A0)1-3}$  and  $C_V^{(A1)1-4}$  therefore equal the coefficients of the seven terms in (19):

$$C_V^{(A0)1} = F_1, \quad C_V^{(A0)2} = F_1 + F_3, \quad C_V^{(A0)3} = F_2, \quad (20)$$

$$\begin{aligned} C_V^{(A1)1} &= F_1 = C_V^{(A0)1}, \\ C_V^{(A1)2} &= F_1 - F_3 = 2C_V^{(A0)1} - C_V^{(A0)2}, \\ C_V^{(A1)3} &= F_1 + F_2/2 = C_V^{(A0)1} + C_V^{(A0)3}/2, \\ C_V^{(A1)4} &= 2F_3 = 2(C_V^{(A0)2} - C_V^{(A0)1}). \end{aligned} \quad (21)$$

We renormalize the SCET operators in the  $\overline{\text{MS}}$  scheme, so the renormalized coefficients follow from the expressions above by cancelling the  $1/\epsilon^2$  and  $1/\epsilon$  poles. The explicit results will be given in Section 4.

The coefficients of the three-body operators cannot be determined from this calculation, because they do not contribute to the  $b \rightarrow q$  matrix element. To extract them we compute the matrix element  $\langle q(p'_1)g(p'_2)|\bar{\psi}\gamma_\mu Q|b(p)\rangle$ , where the gluon is transversely polarized. The Feynman diagrams for this computation are shown in Figure 1. The QCD result of this matrix element must be reproduced in SCET by the expression (of schematic form)

$$\begin{aligned} &\langle T(C^{(A0)}J^{(A0)}, i \int d^4y \mathcal{L}_{\text{int}}^{(0)}) \rangle \\ &+ \{C^{(A1)}\langle J^{(A1)} \rangle + C^{(B1)}\langle J^{(B1)} \rangle\} \\ &+ \langle T(C^{(A1)}J^{(A1)}, i \int d^4y \mathcal{L}_{\text{int}}^{(0)}) \rangle + \langle T(C^{(A0)}J^{(A0)}, i \int d^4y \mathcal{L}_{\text{int}}^{(1)}) \rangle + \mathcal{O}(\lambda^2), \end{aligned} \quad (22)$$



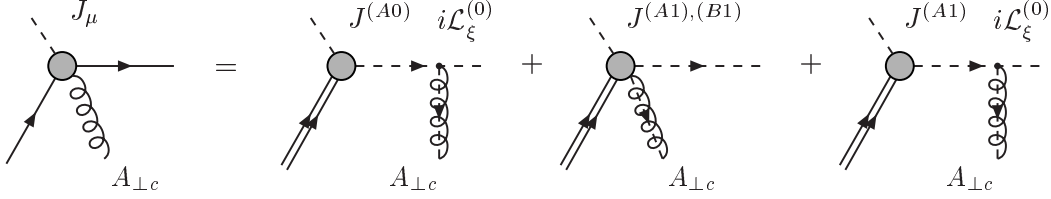


Figure 2: Graphical representation of (22). The shaded circle denotes the current insertion times its short-distance coefficient.

where we have again used that SCET loop diagrams vanish (when expanded in  $\lambda$  in the same way as the QCD diagrams at the level of the Feynman integrands), and we assumed the interaction picture to make the perturbative expansion of the matrix element explicit. This equation is illustrated in Figure 2. It turns out that there is no interaction vertex in the sub-leading Lagrangian  $\mathcal{L}_{\text{int}}^{(1)}$  that could contribute to the  $b \rightarrow qg$  matrix element, when the quark and gluon are both energetic and the gluon is transverse. Therefore the last term in (22) is zero and not shown in the figure.

The diagrams in the first and second row of Figure 1 involve only off-shell propagators when the loop momentum is hard (all components of order  $m_b$ ) and must be reproduced by the “local” terms in the second line of (22). In these diagrams we can immediately drop the small components of the external hard-collinear momenta and set them to  $(n_+ p'_{1,2}) n_- / 2$ . On the other hand, the diagrams in the third row contain nearly on-shell propagators, which makes the extraction of the local contributions less straightforward. The second to fourth diagram in this row have no short-distance contribution, because there is no dependence on an invariant of order  $m_b^2$ . These diagrams can be dropped for the matching calculation. In technical terms, the hard contribution to these diagrams in the sense of an expansion in  $\lambda$  by momentum regions [11, 12] vanishes. The diagrams themselves are non-zero, and correspond to loop contributions to SCET matrix elements. In (22) we already omitted loop contributions to SCET diagrams with the implicit understanding that we compute directly the hard (short-distance) contribution to the diagrams of Figure 1 by expansion of the loop integrand in  $\lambda$ , whenever a hard contribution exists. Otherwise the diagram can be omitted from the matching calculation. This leaves the first diagram in the third row of Figure 1, which we now discuss in some detail.

The QCD computation of this diagram gives

$$D = ig_s \bar{u}(p'_1) \not{A}_{\perp c} \frac{i \not{p}'}{p'^2} \Lambda_\mu(p, p') u(p), \quad (23)$$

where  $p' = p'_1 + p'_2$ , and  $\Lambda_\mu$  denotes the one-particle-irreducible vertex subdiagram. For clarity, we write  $A_\mu$  for the external gluon line with momentum  $p'_2$  rather than  $\epsilon^*(p'_2)$ . The vertex function is decomposed into invariant form factors

$$\Lambda_\mu = \Lambda_\mu^{\text{os}} + \frac{\not{p}'}{m_b} \Lambda_\mu^{\text{off}}$$

$$= F_1 \gamma_\mu + F_2 \frac{p_\mu}{m_b} + F_3 \frac{m_b p'_\mu}{p \cdot p'} + F'_1 \frac{\not{p}'}{m_b} \gamma_\mu + F'_2 \frac{\not{p}'}{m_b} \frac{p_\mu}{m_b} + F'_3 \frac{\not{p}'}{m_b} \frac{m_b p'_\mu}{p \cdot p'} \quad (24)$$

similar to (15) except for three additional terms that vanish for the on-shell vertex function. The form factors are functions of the order-one invariant  $2p \cdot p'/m_b^2$  and the small invariant  $p'^2/m_b^2 \sim \lambda^2$ . Expanding them in  $p'^2/m_b^2$  and inserting the expansion into (23) we find that all but the leading term are already  $\lambda^2$  suppressed. (As remarked above this expansion must be understood as an expansion of the Feynman integrand, and not the expansion of the  $F$ 's after the loop integration.) Hence we compute the form factors by setting  $p'^2 = 0$  from the beginning, and regard them as functions of  $n_- v n_+ p'/m_b$ . In particular,  $F_{1,2,3}$  equal the form factors that appear in (15,20), which determine the coefficients functions of the two-body currents.

To separate the local terms from the non-local time-ordered product terms in this diagram, it proves useful to eliminate  $n_- p'$  through  $n_- p' = (p'^2 - p'^2_\perp)/n_+ p'$  and to write the intermediate hard-collinear propagator as

$$\frac{i \not{p}'}{p'^2} = \left(1 - \frac{\not{n}_+ \not{p}'_\perp}{2 n_+ p'}\right) \frac{i n_+ p'}{p'^2} \frac{\not{n}_-}{2} + \left(\frac{\not{n}_- \not{n}_+}{4} \not{p}'_\perp - \frac{p'^2_\perp \not{n}_+}{n_+ p' 2}\right) \frac{i}{p'^2} + \frac{i}{n_+ p'} \frac{\not{n}_+}{2}. \quad (25)$$

Inserting this into (23) we find, make use of (17, 24),

$$\begin{aligned} D = & ig_s \bar{u}_c(p'_1) \left[ \not{A}_{\perp c} \frac{\not{p}'_\perp}{n_+ p'} + \frac{\not{p}'_{1\perp}}{n_+ p'_1} \not{A}_{\perp c} \right] \frac{\not{n}_+ i n_+ p'}{p'^2} \frac{\not{n}_-}{2} \left(1 - \frac{\not{p}'_\perp \not{n}_+}{n_+ p' 2}\right) \Lambda_\mu^{\text{os}} h_v \\ & - g_s \bar{u}_c(p'_1) \frac{\not{A}_{\perp c}}{m_b} \left( \frac{m_b}{n_- v n_+ p'} \not{n}_- \frac{\not{n}_+}{2} \Lambda_\mu^{\text{os}} + \Lambda_\mu^{\text{off}} \right) u_v. \end{aligned} \quad (26)$$

The first line gives *precisely* the (non-zero) time-ordered product terms  $\langle T(C^{(A0)} J^{(A0)} + C^{(A1)} J^{(A1)}, i \int d^4 y \mathcal{L}_{\text{int}}^{(0)}) \rangle$  in (22). Subtracting these terms, we obtain the following result for the local contribution ( $x = n_- v n_+ p'/m_b$ )

$$\begin{aligned} D_{\text{local}} = & \left( \frac{F_1}{x} - F'_1 \right) \bar{u}_c(p'_1) \frac{g_s \not{A}_{\perp c}}{m_b} \gamma_{\perp \mu} u_v - \left( \frac{F_1 - F_3}{x} - F'_1 - F'_3 \right) \bar{u}_c(p'_1) \frac{g_s \not{A}_{\perp c}}{m_b} \frac{(-n_- \mu)}{n_- v} u_v \\ & - \left( \frac{2F_1 + F_2}{x} + F'_2 \right) \bar{u}_c(p'_1) \frac{g_s \not{A}_{\perp c}}{m_b} v_\mu u_v, \end{aligned} \quad (27)$$

which must be matched to  $C^{(A1)} \langle J^{(A1)} \rangle + C^{(B1)} \langle J^{(B1)} \rangle$ .

Combining this with the seven diagrams from the first two rows of Figure 1, the counterterm diagrams and the on-shell residue factors in the  $\overline{\text{MS}}$  scheme, we determine the unrenormalized short-distance coefficient functions by comparing the coefficients of the various spinor structures. For instance, focussing on the  $\bar{u}_c(p') g_s \not{A}_{\perp c} / m_b v_\mu u_v$  structure, calling its coefficient  $T_V^3$ , we obtain

$$T_V^3 = C_V^{(B1)3} - 2 C_V^{(A1)3}. \quad (28)$$

Since we have already determined the  $C_V^{(A1)k}$ , this provides the required result for the “B1” coefficients. Renormalization in the effective theory involves the operator renormalization constant and the renormalization constant related to the coupling  $g_s$ . We adopt the same definition of the renormalized coupling in QCD and in SCET, so the coupling renormalization factors cancel in the matching. Operator renormalization in the  $\overline{\text{MS}}$  scheme amounts to cancelling the remaining  $1/\epsilon$  poles. The heavy quark mass is defined in the pole scheme. Furthermore, we assume the NDR scheme for  $\gamma_5$ , where  $\gamma_5$  anti-commutes with  $\gamma_\mu$ .

## 4 Results for the coefficient functions

The matching calculation proceeds the same way for any Dirac structure of the QCD weak current. In this section we summarize the results.

### 4.1 Two-body operators

*Scalar current*

$$C_S^{(A0)}(x) = 1 - \frac{\alpha_s C_F}{4\pi} \left[ -6 \ln\left(\frac{\nu}{m_b}\right) + 2 \ln^2\left(\frac{\mu}{m_b}\right) - (4 \ln x - 5) \ln\left(\frac{\mu}{m_b}\right) + 2 \ln^2 x + 2 \text{Li}_2(1-x) + \frac{\pi^2}{12} - \frac{2 \ln x}{1-x} \right] \quad (29)$$

$$C_S^{(A1)}(x) = C_S^{(A0)}(x) \quad (30)$$

Recall that  $x = n_- v n_+ p' / m_b$ ,  $\nu$  is the renormalization scale of the QCD weak current, and  $\mu$  is the SCET renormalization scale. The  $\mu$  dependence cancels the dependence of the SCET current operators on  $\mu$ .

*Pseudo-scalar current*

The coefficients  $C_P^{(A0)}(x)$ ,  $C_P^{(A1)}(x)$  for the pseudo-scalar current coincide with the corresponding scalar ones.

*Vector current*

$$C_V^{(A0)1}(x) = 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln^2\left(\frac{\mu}{m_b}\right) - (4 \ln x - 5) \ln\left(\frac{\mu}{m_b}\right) + 2 \ln^2 x + 2 \text{Li}_2(1-x) + \frac{\pi^2}{12} + \left(\frac{1}{1-x} - 3\right) \ln x + 6 \right] \quad (31)$$

$$C_V^{(A0)2}(x) = 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln^2\left(\frac{\mu}{m_b}\right) - (4 \ln x - 5) \ln\left(\frac{\mu}{m_b}\right) + 2 \ln^2 x + 2 \text{Li}_2(1-x) + \frac{\pi^2}{12} + \left(\frac{x^2}{(1-x)^2} - 2\right) \ln x + \frac{x}{1-x} + 6 \right] \quad (32)$$

$$C_V^{(A0)3}(x) = \frac{\alpha_s C_F}{4\pi} \left[ \frac{2x}{(1-x)^2} \ln x + \frac{2}{1-x} \right] \quad (33)$$

$$C_V^{(A1)1}(x) = C_V^{(A0)1}(x) \quad (34)$$

$$C_V^{(A1)2}(x) = 2C_V^{(A0)1}(x) - C_V^{(A0)2}(x) \quad (35)$$

$$C_V^{(A1)3}(x) = C_V^{(A0)1}(x) + C_V^{(A0)3}(x)/2 \quad (36)$$

$$C_V^{(A1)4}(x) = -2C_V^{(A0)1}(x) + 2C_V^{(A0)2}(x) \quad (37)$$

### *Axial current*

The axial current coefficients are related to those of the vector current by  $C_A^{(A0)1,2}(x) = C_V^{(A0)1,2}(x)$ ,  $C_A^{(A0)3}(x) = -C_V^{(A0)3}(x)$ ,  $C_A^{(A1)1-3}(x) = C_V^{(A1)1-3}(x)$ ,  $C_A^{(A1)4}(x) = -C_V^{(A1)4}(x)$ .

### *Tensor current*

$$C_T^{(A0)1}(x) = 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln \left( \frac{\nu}{m_b} \right) + 2 \ln^2 \left( \frac{\mu}{m_b} \right) - (4 \ln x - 5) \ln \left( \frac{\mu}{m_b} \right) + 2 \ln^2 x + 2 \text{Li}_2(1-x) + \frac{\pi^2}{12} + \left( \frac{2}{1-x} - 4 \right) \ln x + 6 \right] \quad (38)$$

$$C_T^{(A0)2}(x) = 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln \left( \frac{\nu}{m_b} \right) + 2 \ln^2 \left( \frac{\mu}{m_b} \right) - (4 \ln x - 5) \ln \left( \frac{\mu}{m_b} \right) + 2 \ln^2 x - 2 \ln x + 2 \text{Li}_2(1-x) + \frac{\pi^2}{12} + 6 \right] \quad (39)$$

$$C_T^{(A0)3}(x) = 0 \quad (40)$$

$$C_T^{(A0)4}(x) = C_T^{(A0)1}(x) \quad (41)$$

$$C_T^{(A1)1}(x) = 2C_T^{(A0)1}(x) - C_T^{(A0)2}(x) \quad (42)$$

$$C_T^{(A1)2}(x) = C_T^{(A0)1}(x) + C_T^{(A0)3}(x)/2 \quad (43)$$

$$C_T^{(A1)3}(x) = 2C_T^{(A0)2}(x) + 2C_T^{(A0)3}(x) - C_T^{(A0)4}(x) \quad (44)$$

$$C_T^{(A1)4}(x) = -C_T^{(A0)1}(x) + 2C_T^{(A0)2}(x) \quad (45)$$

$$C_T^{(A1)5}(x) = -2C_T^{(A0)1}(x) + 2C_T^{(A0)2}(x) \quad (46)$$

$$C_T^{(A1)6}(x) = 2C_T^{(A0)1}(x) + 2C_T^{(A0)3}(x) - 2C_T^{(A0)4}(x) \quad (47)$$

$$C_T^{(A1)7}(x) = C_T^{(A0)1}(x) \quad (48)$$

We note that  $C_T^{(A1)6}(x) = 0$  up to the one-loop order.

The coefficients for the leading-power (“A0”) currents have been computed [3] in the same factorization scheme as adopted here. The results above are in agreement with the previous calculation.

## 4.2 Three-body operators

*Scalar current*

$$\begin{aligned}
C_S^{(B1)} = & \frac{\alpha_s C_F}{4\pi} \left[ \left( \frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) - \frac{4}{x} \right) \ln \left( \frac{\mu}{m_b} \right) - \frac{2}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) \right. \\
& + \left( \frac{4}{x} + \frac{2}{1-x} \right) \ln x - \frac{2}{x_1} \left( \frac{\ln x}{1-x} - \frac{\ln x_2}{1-x_2} \right) - \frac{x_2 \ln x_2}{(1-x_2)^2} \\
& + \frac{2(1-x_1)}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) - \frac{2}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \\
& \left. - \frac{4}{x} - \frac{1}{1-x_2} \right] \\
& - \frac{\alpha_s C_A}{4\pi} \left[ \frac{2}{x_2} \ln \left( \frac{x}{x_1} \right) \ln \left( \frac{\mu}{m_b} \right) - \frac{1}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) + \frac{1}{x_1} \ln \left( \frac{x}{x_2} \right) \right. \\
& \left. + \frac{\ln x_2}{1-x_2} + \frac{1-x_1}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) - \frac{1}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \right]
\end{aligned} \tag{49}$$

Here  $x_i = n_- v n_+ p'_i / m_b$ ,  $x = x_1 + x_2$ . The  $\mu$ -dependent terms show that the two-body and three-body operators mix under renormalization.

*Pseudo-scalar current*

For the pseudo-scalar current we find  $C_P^{(B1)}(x_1, x_2) = -C_S^{(B1)}(x_1, x_2)$ .

*Vector current*

$$\begin{aligned}
C_V^{(B1)1} = & \frac{\alpha_s C_F}{4\pi} \left[ \left( \frac{4}{x} - \frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) \right) \ln \left( \frac{\mu}{m_b} \right) + \frac{2}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) \right. \\
& + \frac{2}{x_1} \ln \left( \frac{x}{x_2} \right) - \frac{4}{x} \ln x + \frac{x_2 \ln x_2}{(1-x_2)^2} - \frac{2(1-x)}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) \\
& \left. + \frac{2(1-x_2)}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) + \frac{4}{x} + \frac{1}{1-x_2} \right] \\
& - \frac{\alpha_s C_A}{4\pi} \left[ -\frac{2}{x_2} \ln \left( \frac{x}{x_1} \right) \ln \left( \frac{\mu}{m_b} \right) + \frac{1}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) - \frac{1}{x_1} \ln \left( \frac{x}{x_2} \right) \right. \\
& \left. - \frac{\ln x_2}{1-x_2} - \frac{1-x}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) + \frac{1-x_2}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \right]
\end{aligned} \tag{50}$$

$$\begin{aligned}
C_V^{(B1)2} = & 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln^2 \left( \frac{\mu}{m_b} \right) - \left( 4 \ln x - \frac{4x}{x_2} \ln \left( \frac{x}{x_1} \right) - 1 \right) \ln \left( \frac{\mu}{m_b} \right) + \frac{4}{x} \ln \left( \frac{\mu}{m_b} \right) \right. \\
& - \frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) \ln \left( \frac{\mu}{m_b} \right) + 2 \ln^2 x - \left( \frac{4}{x} + \frac{1}{(1-x)^2} + 1 \right) \ln x + 2 \text{Li}_2(1-x) \\
& \left. + \frac{2(1-x)}{x_2} \left( \ln^2 x - \ln^2 x_1 \right) - \frac{6-4x}{x_2} \ln \left( \frac{x}{x_1} \right) - \frac{1}{x_1} \ln \left( \frac{x}{x_2} \right) - \frac{(1-2x) \ln x}{x_1 (1-x)^2} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{(1-2x_2)\ln x_2}{x_1(1-x_2)^2} + \left( \frac{2(1-x)}{x_2} + \frac{2}{x_1x_2} \right) \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) \\
& - \frac{2}{x_1x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) + \frac{x_2}{x_1} \left( \frac{1}{1-x} - \frac{1}{1-x_2} \right) + \frac{4}{x} + \frac{\pi^2}{12} + 1 \Big] \\
& + \frac{\alpha_s C_A}{4\pi} \left[ - \frac{2(1-x)}{x_2} \ln \left( \frac{x}{x_1} \right) \ln \left( \frac{\mu}{m_b} \right) + \frac{1-x}{x_2} \left( \ln^2 x - \ln^2 x_1 \right) \right. \\
& + \frac{2x-3}{x_2} \ln \left( \frac{x}{x_1} \right) - \frac{2-x}{x_1} \ln \left( \frac{x}{x_2} \right) - \frac{1}{x_1} \left( \frac{\ln x}{1-x} - \frac{\ln x_2}{1-x_2} \right) \\
& \left. + \left( \frac{1-x}{x_2} + \frac{1}{x_1x_2} \right) \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) - \frac{1}{x_1x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \right] \quad (51)
\end{aligned}$$

$$\begin{aligned}
C_V^{(B1)3} = & \frac{\alpha_s C_F}{4\pi} \left[ \left( \frac{8}{x_2} \ln \left( \frac{x}{x_1} \right) - \frac{8}{x} \right) \ln \left( \frac{\mu}{m_b} \right) - \frac{4}{x_2} \left( \ln^2 x - \ln^2 x_1 \right) + \frac{8}{x_2} \ln \left( \frac{x}{x_1} \right) \right. \\
& + \left( \frac{8}{x} + \frac{4}{1-x} \right) \ln x - \frac{2x_2}{x_1} \left( \frac{\ln x}{(1-x)^2} - \frac{\ln x_2}{(1-x_2)^2} \right) \\
& - \frac{4}{x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) - \frac{8}{x} - \frac{2x_2}{(1-x_2)(1-x)} \Big] \\
& - \frac{\alpha_s C_A}{4\pi} \left[ \frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) \ln \left( \frac{\mu}{m_b} \right) - \frac{2}{x_2} \left( \ln^2 x - \ln^2 x_1 \right) + \frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) \right. \\
& \left. + \frac{2}{x_1} \left( \frac{(2-x)\ln x}{1-x} - \frac{(2-x_2)\ln x_2}{1-x_2} \right) - \frac{2}{x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) \right] \quad (52)
\end{aligned}$$

$$\begin{aligned}
C_V^{(B1)4} = & \frac{\alpha_s C_F}{4\pi} \left[ \frac{2\ln x}{(1-x)^2} + \frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) + \frac{2}{x_1} \left( \frac{(2-x)\ln x}{1-x} - \frac{(2-x_2)\ln x_2}{1-x_2} \right) - \frac{4\ln x}{1-x} \right. \\
& + \frac{4\ln x_1}{1-x_1} - \frac{2x_2\ln x_2}{(1-x_2)^2} - \frac{4}{x_1x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) - \text{Li}_2(1-x_2) + \frac{\pi^2}{6} \right) \\
& \left. + \frac{2}{1-x} - \frac{2}{1-x_2} \right] \\
& - \frac{\alpha_s C_A}{4\pi} \left[ \frac{2}{x_2} \ln \left( \frac{x}{x_1} \right) + \frac{2}{x_1} \ln \left( \frac{x}{x_2} \right) + \frac{2\ln x_1}{1-x_1} + \frac{2\ln x_2}{1-x_2} \right. \\
& \left. - \frac{2}{x_1x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) - \text{Li}_2(1-x_2) + \frac{\pi^2}{6} \right) \right] \quad (53)
\end{aligned}$$

*Axial current*

The axial current coefficients are related to those of the vector current by  $C_A^{(B1)1,4}(x) = -C_V^{(B1)1,4}(x)$  and  $C_A^{(B1)2,3}(x) = C_V^{(B1)2,3}(x)$ .

Tensor current

$$\begin{aligned}
C_T^{(B1)1} = & 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln \left( \frac{\nu}{m_b} \right) + 2 \ln^2 \left( \frac{\mu}{m_b} \right) - \left( 4 \ln x_1 - \frac{4x}{x_2} \ln \left( \frac{x}{x_1} \right) - 1 \right) \ln \left( \frac{\mu}{m_b} \right) \right. \\
& + \left( \frac{4}{x} - \frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) \right) \ln \left( \frac{\mu}{m_b} \right) + 2 \ln^2 x - \frac{4}{x} \ln x + 2 \text{Li}_2(1-x) + \frac{4}{x} + \frac{\pi^2}{12} + 3 \\
& + \frac{2(1-x_1-2x_2)}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) - \left( \frac{2x}{x_1} - \frac{2}{x_1(1-x)} \right) \ln x \\
& + \left( \frac{2x_2}{x_1} - \frac{2}{x_1(1-x_2)} \right) \ln x_2 + \frac{2(1-x_2)}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \\
& + \left. \left( \frac{2(2-x_1-2x_2)}{x_2} - \frac{2(1-x_2)}{x_1 x_2} \right) \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) \right] \quad (54) \\
& + \frac{\alpha_s C_A}{4\pi} \left[ \left( \frac{2x_2}{x_1} \ln \left( \frac{x}{x_2} \right) - \frac{2(1-x_1-2x_2)}{x_2} \ln \left( \frac{x}{x_1} \right) \right) \ln \left( \frac{\mu}{m_b} \right) \right. \\
& - \frac{x_2}{x_1} \left( \ln^2 x - \ln^2 x_2 \right) + \frac{1-x_1-2x_2}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) \\
& - \frac{1-x_1-2x_2}{x_1} \ln \left( \frac{x}{x_2} \right) + \frac{2-x_1-2x_2}{x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) \\
& - \frac{x_2}{x_1} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_2) \right) \\
& - \left. \frac{1-x_2}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) - \text{Li}_2(1-x_2) + \frac{\pi^2}{6} \right) + 1 \right]
\end{aligned}$$

$$\begin{aligned}
C_T^{(B1)2} = & \frac{\alpha_s C_F}{4\pi} \left[ \left( \frac{8}{x} - \frac{8}{x_2} \ln \left( \frac{x}{x_1} \right) \right) \ln \left( \frac{\mu}{m_b} \right) + \frac{4}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) \right. \\
& - \left( \frac{8}{x} + \frac{4}{1-x} \right) \ln x + \frac{4}{x_1} \left( \frac{\ln x}{1-x} - \frac{\ln x_2}{1-x_2} \right) \\
& - \frac{4(1-x_1)}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) + \frac{4}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) + \frac{8}{x} \quad (55) \\
& - \frac{\alpha_s C_A}{4\pi} \left[ -\frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) \ln \left( \frac{\mu}{m_b} \right) + \frac{2}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) - \frac{2}{x_1} \ln \left( \frac{x}{x_2} \right) \right. \\
& - \left. \frac{2(1-x_1)}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) + \frac{2}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \right]
\end{aligned}$$

$$\begin{aligned}
C_T^{(B1)3} = & \frac{\alpha_s C_F}{4\pi} \left[ \left( \frac{4}{x_2} \ln \left( \frac{x}{x_1} \right) - \frac{4}{x} \right) \ln \left( \frac{\mu}{m_b} \right) - \frac{2}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln \left( \frac{x}{x_1} \right) \right) \right. \\
& + \left( \frac{4}{x} + \frac{2}{1-x} - \frac{2}{x_1(1-x)} \right) \ln x + \left( \frac{2}{x_1(1-x_2)} + \frac{x_2}{(1-x_2)^2} \right) \ln x_2 \\
& + \left. \frac{2(1-x_1)}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) - \frac{2}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{1-x_2} - \frac{4}{x} \Big] \\
& - \frac{\alpha_s C_A}{4\pi} \left[ \frac{2}{x_2} \ln\left(\frac{x}{x_1}\right) \ln\left(\frac{\mu}{m_b}\right) - \frac{1}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln\left(\frac{x}{x_1}\right) \right) + \frac{1}{x_1} \ln\left(\frac{x}{x_2}\right) \right. \\
& \left. - \frac{\ln x_2}{1-x_2} + \frac{1-x_1}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) - \frac{1}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \right]
\end{aligned} \tag{56}$$

$$C_T^{(B1)4} = -C_T^{(B1)3} \tag{57}$$

$$\begin{aligned}
C_T^{(B1)5} = & 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln\left(\frac{\nu}{m_b}\right) + 2 \ln^2\left(\frac{\mu}{m_b}\right) - \left( 4 \ln x_1 - \frac{4x}{x_2} \ln\left(\frac{x}{x_1}\right) - 1 \right) \ln\left(\frac{\mu}{m_b}\right) \right. \\
& + 2 \ln^2 x + \frac{2x \ln x}{1-x} + 2 \text{Li}_2(1-x) + \frac{\pi^2}{12} + 3 - \frac{2(x_1+2x_2)}{x_2} \left( \ln^2 x - \ln^2 x_1 \right) \\
& + \frac{2x}{x_2} \ln\left(\frac{x}{x_1}\right) - \frac{2x_2}{x_1} \ln\left(\frac{x}{x_2}\right) - \frac{2}{x_1} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \\
& + \left( \frac{2}{x_1} + \frac{2(1-x_1-2x_2)}{x_2} \right) \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) \Big] \\
& + \frac{\alpha_s C_A}{4\pi} \left[ \left( \frac{2x}{x_2} \ln\left(\frac{x}{x_1}\right) + \frac{2x}{x_1} \ln\left(\frac{x}{x_2}\right) - 2 \ln\left(\frac{x_1}{x_2}\right) \right) \ln\left(\frac{\mu}{m_b}\right) + \left( \ln^2 x_1 - \ln^2 x_2 \right) \right. \\
& - \frac{x}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln\left(\frac{x}{x_1}\right) \right) - \frac{x}{x_1} \left( \ln^2 x - \ln^2 x_2 - \ln\left(\frac{x}{x_2}\right) \right) \\
& + \frac{1}{x_1} \left( (1-x) \text{Li}_2(1-x) - (1-x_1) \text{Li}_2(1-x_1) - (1-x_2) \text{Li}_2(1-x_2) + \frac{\pi^2}{6} \right) \\
& \left. + \frac{1-x}{x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) + 1 \right]
\end{aligned} \tag{58}$$

$$C_T^{(B1)6} = 0 \tag{59}$$

$$\begin{aligned}
C_T^{(B1)7} = & \frac{\alpha_s C_F}{4\pi} \left[ \left( \frac{4}{x} - \frac{4}{x_2} \ln\left(\frac{x}{x_1}\right) \right) \ln\left(\frac{\mu}{m_b}\right) + \frac{2}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln\left(\frac{x}{x_1}\right) \right) \right. \\
& + \frac{4}{x_1} \ln\left(\frac{x}{x_2}\right) - \left( \frac{4}{x} - \frac{2}{1-x} \right) \ln x - \frac{2}{x_1} \left( \frac{\ln x}{1-x} - \frac{\ln x_2}{1-x_2} \right) + \frac{x_2 \ln x_2}{(1-x_2)^2} \\
& - \frac{2(1-x_1-2x_2)}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) + \frac{2(1-2x_2)}{x_1 x_2} \left( \text{Li}_2(1-x_2) - \frac{\pi^2}{6} \right) \\
& \left. + \frac{4}{x} + \frac{1}{1-x_2} \right] \\
& - \frac{\alpha_s C_A}{4\pi} \left[ - \frac{2}{x_2} \ln\left(\frac{x}{x_1}\right) \ln\left(\frac{\mu}{m_b}\right) + \frac{1}{x_2} \left( \ln^2 x - \ln^2 x_1 - \ln\left(\frac{x}{x_1}\right) \right) - \frac{1}{x_1} \ln\left(\frac{x}{x_2}\right) \right. \\
& \left. - \frac{\ln x_2}{1-x_2} - \frac{1-x_1-2x_2}{x_1 x_2} \left( \text{Li}_2(1-x) - \text{Li}_2(1-x_1) \right) \right]
\end{aligned} \tag{60}$$



$$+ \frac{1 - 2x_2}{x_1 x_2} \left( \text{Li}_2(1 - x_2) - \frac{\pi^2}{6} \right) \Big]$$

## 5 Conclusion

In this paper we computed the one-loop (hard) matching correction to heavy-to-light transition currents in soft-collinear effective theory (SCET) to sub-leading power in the SCET expansion parameter  $\lambda$  for an arbitrary Dirac structure of the QCD weak current.

The phenomenological applications of this result require further calculations, which we expect to be completed in the future. To give two examples, we recall that heavy-to-light transition form factors become simpler at large momentum transfer [13]. For instance, the three different form factors for  $B \rightarrow \pi$  factorize as [6]

$$f_i(E_\pi) = C_i^{(A0)}(E_\pi) \xi_\pi(E_\pi) + \int_0^\infty \frac{d\omega}{\omega} \int_0^1 du T_i(E_\pi; \ln \omega, u) \phi_{B+}(\omega) \phi_\pi(u), \quad (61)$$

where  $\xi_\pi(E_\pi)$  is a single non-perturbative form factor, and  $\phi_X$  are light-cone distribution amplitudes. The hard-scattering kernels  $T_i$  are convolutions of hard coefficient functions with hard-collinear coefficient functions,  $T_i \sim \sum_k C_i^{(1)} \star J_k$  [7, 8, 14]. The calculation reported in this paper completes the hard part of the next-to-leading order result for the hard-scattering kernels, since the  $C_i^{(1)}$  are expressed in terms of  $C_X^{(A1)k}$  and  $C_X^{(B1)k}$ . (We refrain from giving a numerical result for the hard contribution alone, since it depends on the factorization scheme, such that only the product with the hard-collinear coefficient acquires a physical meaning, given a definition of the light-cone distribution amplitudes.)

Second, SCET offers the possibility to extend the factorization theorems for semi-inclusive heavy quark decays to sub-leading order in  $1/m_b$ . The relevant quantity here is the product of two weak transition currents. The double insertions of  $\lambda$ -suppressed currents operators are one ingredient in this calculation, which can be obtained straightforwardly from the above results. Here a complete result at order  $\alpha_s/m_b$  requires also the interference of the leading-power currents with the  $\lambda^2$ -suppressed operators.

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